Statistical Neurodynamics of Deep Networks: Signal Transformations and Fisher Information

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Abstract—Statistical neurodynamics studies macroscopic behaviors of randomly connected neural networks. We consider feedforward deep layered networks where input signals are processed layer by layer. The manifold of input signals is embedded in a higher-dimensional manifold as a curved submanifold or reduced to a lower-dimensional manifold. We prove that the manifold is transformed conformally, keeping the original shape. We study the enlargement ratio and curvature of the embedded manifold. The distance between two signals changes, and eventually converges to a constant, provided the number of neurons and the number of layers are infinitely large. In reality, they are finite so there remain frustrations. We study the effect of finiteness.

Considering a deep network as a stochastic machine transforming input signals to output signals. we can discuss the Fisher information matrix of the network parameters. We prove that the Fisher information matrix is nearly block-diagonal except for small off-diagonal elements. We further show that a block corresponding to each unit is nearly diagonal except for bias-weight interaction terms. This makes it easy to implement natural gradient learning without matrix inversion. This justifies the quasi-diagonal natural gradient proposed by Yann Ollivier.

Bio—Shun-ichi Amari was born in 1936, graduated from the U. Tokyo in 1958, worked in Kyushu University and U Tokyo, and now its professor-emeritus. He also worked in RIKEN Brain Science Institute (director in 2003- 2008) and is now a honorary advisor of RIKEN. He served as Presidents of INNS and IEICE, Japan. He is recipients of IEEE Piore Award, IEEE Neural Networks Pioneer Award, Gabor Award (INNS), Japan Academy Award and Order of Cultural Merit of Japan, among many others. He has retired, but is still working on mathematical research as his hobby, as well as playing go-game.